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Adiabatic Spherical Shock Waves in Rotating Magnetized Ideal Gas: Weak-Field Approximation

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Abstract: *Adiabatic propagation of spherical magneto hydrodynamic shock waves in an ideal gas has been carried out by the well-known CCW method under the impact of solid body rotation of medium. The effect of constant axial weak magnetic field of the phenomenon is estimated for the converging strong shock with constant angular velocity. Neglecting the flow behind the shock front the analytical expressions for shock velocity, shock strength, pressure and particle velocity just behind the shock front have been derived and discussed with the graphs and tables. The results accomplished here have been compared with dose for normal shock wave and earlier results.*

Key Words: *Magneto hydrodynamic, strong, shock, weak magnetic field, CCW theory*

I.INTRODUCTION

The study of production, propagation and attenuation of shock waves produced by explosions on the ground and deep sea are of great importance in war faze propagation of hydrodynamic shock wave has in the recent part received considerable attention of many workers. Kumar [1], Vshwakarma [2], Singh and Pandey [3] are among them. Using Chester[4]-Chisnell[5]-Whitham[6] method, Prakash and Kumar[7] have studied the freely propagation of diverging plane and cylindrical shock waves in an ideal gas in presence of an axial magnetic field. Further, the correction to the CCW method has been proposed by Yousaf[8] , Chisnell and Yousaf[9] and Yadav[10] . Yadav and Gangwar [11] have investigated the motion of spherical converging strong shock for freely as well as under the influence of overtaking disturbances. Gangwar et al. [12] have investigate the effect of weak magnetic field on motion of strong converging shock wave in an ideal gas having a power varying density distribution under the influence of self-gravitation. Recently Gangwar[13] have investigate the effect of solid dust particles on the motion of strong shock in non-ideal gaseous medium by CCW theory under the influence of overtaking waves.

The aim of the present paper, is to study the propagation of strong spherical hydromagnetic shock in a rotating gas in presence of constant axial magnetic field under weak field approximation. The effect of overtaking disturbances on the motion of shock is assumed to be negligible. Assuming the non- uniformity of the medium as $\rho o=\rho' r^{-\omega}$ where ρ' is density at the centre and ω is a constant. The non-uniformity of the medium arises due to solid body rotation of gas. The magnetic field is taken to be axial and initially of constant strength. Analytical expressions for shock velocity, shock strength have been derived, simultaneously, for both cases. Expressions for the pressure and particle velocity have also been obtained. The variation of these parameters with propagation distance (r), adiabatic index (γ), magnetic field (β) and density parameter (ω), has been numerically calculated and displayed in figures. The results accomplished here are compared with earlier available results obtained by Yadav and Gangwar[11] .

II. BASIC EQUATIONS, BOUNDARY CONDITIONS AND ANALYTICAL EXPRESSIONS:

The equations governing the spherically symmetrical flow enclosed by the shock front are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{2\rho}\frac{\partial H^2}{\partial r} - \frac{v^2}{r} = 0 \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \tag{2}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\left(p\rho^{-\gamma}\right) = 0 \tag{3}$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(vr) = 0 \tag{5}$$

where u, v are the radial and azimuthal components of particle velocity, p and ρ denote, respectively, the pressure and density at distance r from the origin at time t, and γ is the adiabatic index of gas.

The magenetohydrodynamic shock conditions can be written in terms of a single parameter $\xi = \rho/\rho_o$ as

$$\rho = \rho_0\xi, \qquad H = H_0\xi, \qquad u = \frac{(\xi-1)}{\xi}U$$

$$U^2 = \frac{2\xi}{(\gamma+1)-(\gamma-1)\xi}\left[a_0^2 + \frac{b_0^2}{2}\{(2-\gamma)\xi+\gamma\}\right]$$

$$p = p_0 + \frac{2\rho_0(\xi-1)}{(\gamma+1)-(\gamma-1)\xi}\left[a_0^2 + \frac{(\gamma-1)}{4}b_0^2(\xi-1)^2\right] \tag{6}$$

where suffix "$_o$" stands for the state immediately ahead of the shock, U is the shock velocity, a_0 is the speed of sound ($\sqrt{\gamma p_0/\rho_0}$) in unperturbed medium and b_0 is the Alfven speed ($\sqrt{\mu H_0^2/\rho_0}$). For weak magnetic $\left(b_0^2 << a_0^2\right)$ field using this condition, the boundary conditions(6) become

$$\rho = \rho_0\xi, \qquad H = H_0\xi, \qquad u = \frac{(\xi-1)}{\xi}U, \qquad \frac{p}{p_0} = 1 + \left(\chi' a_0^2 + N_1 b_0^2\right)\frac{U^2}{a_0^4} \tag{7}$$

where χ' and F_1 are the constant taken as

$$\chi' = \frac{\gamma(\xi-1)}{\xi}, \qquad F_1 = \frac{\gamma(\xi-1)}{4\xi}\left[(\gamma-1)(\xi-1)^2 - 2\{(2-\gamma)\xi+\gamma\}\right]$$

For spherical converging shocks, the characteristic form of the system of equations(10-(2)- , is

$$\mathbf{dp - \rho c du + \mu H dH + \frac{2\rho c^2 u}{(u-c)}\frac{dr}{r} + \frac{\rho c v^2}{(u-c)}\frac{dr}{r} = 0} \tag{8}$$

Here $c^2 = a^2 + b^2 = \left(\gamma p + \mu H^2\right)/\rho$.

The condition of hydrostatic equilibrium prevailing the front of the shock is written by substituting $\partial/\partial t = 0 = u$ and H_o =constant and $v = r\Omega_0^2$, where Ω_0 is the initial angular velocity of rotating gas, in the above equation. Consequently, the equilibrium condition prevailing the front of the shock can be written as

$$\frac{1}{\rho_0}\frac{dp_0}{dr} - r\Omega_0^2 = 0 \tag{1}$$

Assuming that the medium in which shock propagates is at rest and initial density distribution is given by

$$\rho_0 = \rho' r^{-\omega} \tag{2}$$

where, ρ' is the density at centre and ω is a constant. On solving equation(1), we get

$$\Rightarrow \qquad p_0 = \frac{\Omega_0^2 \rho' r^{2-\omega}}{(2-\omega)} \tag{11}$$

For positive finite pressure, as defined by equation(11), requires that ω should obey the inequality as ω<2. Using equation(10) and (11), we get

$$\therefore \qquad a_0 = r\Omega_0 \sqrt{\frac{\gamma}{(2-\omega)}} \tag{12}$$

we know that

$$b_0^2 = \frac{\mu H_0^2}{\rho_0} \tag{13}$$

Using the conditions(9)-(13), and using $c = U\sqrt{\chi'/\xi}$, and putting $dH_o = 0$ (since H_o = constant) in equation(8) and after simplifying , we get

$$\Rightarrow \quad dU^2 + \frac{1}{F_2}\left[1 - \frac{F_1 b_0^2}{F_2 \gamma a_0^2}\right]\left[\frac{\chi' dp_0}{\gamma p_0} + \frac{F_1 b_0^2 dp_0}{\gamma p_0 a_0^2} - \frac{2\chi' da_0}{\gamma a_0} + \frac{2F_1 b_0 db_0}{\gamma a_0^2} - \frac{4F_1 b_0^2 da_0}{\gamma a_0^3}\right.$$
$$\left. + \frac{2\chi'(\xi-1)}{(\xi-1)-\sqrt{\chi'\xi}}\frac{dr}{r^2}\right]U^2 + \frac{1}{F_2}\left[1 - \frac{F_1 b_0^2}{F_2\gamma a_0^2}\right]\left[\frac{dp_0}{\rho_0} + \frac{\xi\sqrt{\chi'\xi}}{(\xi-1)-\sqrt{\chi'\xi}}\Omega_0^2 r dr\right] = 0 \tag{14}$$

where $F_2 = \frac{\chi'}{\gamma} - \frac{(\xi-1)}{2}\sqrt{\frac{\chi'}{\xi}}$ and $\left[\frac{p_0 \chi' F_1}{F_2 \mu H_0^2} < 1\right]$

or

$$\frac{dU^2}{dr} + \left[\frac{J_1}{r} + \frac{J_2 \beta^2 r^{\omega-3}}{\Omega_0^2}\right]U^2 = -J_3\Omega_0^2 r + J_4 r^{\omega-1} \tag{15}$$

where $J_1 = \frac{\chi'}{F_2}\left[\frac{2(\xi-1)}{(\xi-1)-\sqrt{\chi'\xi}} - \frac{\omega}{\gamma}\right]$, $J_2 = \frac{F_1 a'^2(2-\omega)}{F_2\gamma^2}[2-J_1]$, $J_3 = \frac{1}{F_2}\left[1 + \frac{\xi\sqrt{\chi'\xi}}{(\xi-1)-\sqrt{\chi'\xi}}\right]$, $J_4 = \frac{F_1 a'^2 J_3}{F_2 \gamma^2}$

Now solving equation(15), we get

$$r^{J_1}\exp\left\{\frac{J_2\beta^2 r^{\omega-2}}{(\omega-2)\Omega_0^2}\right\}U^2 = -\int J_3\Omega_0^2 r^{J_1+1}dr + \int\frac{J_2 J_3 \beta^2 r^{J_1+\omega-1}}{\omega-2}dr + \int J_4\beta^2 r^{J_1+\omega-1}dr + k_2$$

where k_2 is constant of integration.

Expressions for shock velocity (U) and shock strength (U/a_o) for strong spherical converging shock in presence of weak magnetic field can be written a

$$\mathbf{U = \left[k_2 r^{-J_1} - \frac{J_3 r^2 \Omega_0^2}{J_1+2} - \frac{J_2 J_3 \beta^2 r^{\omega}}{(\omega-2)(J_1+\omega)} + \frac{J_4\beta^2 r^{\omega}}{(J_1+\omega)}\right]^{1/2} \exp\left\{\frac{J_2\beta^2 r^{\omega-2}}{2(2-\omega)\Omega_0^2}\right\}} \tag{16}$$

$$\mathbf{\frac{U}{a_0} = \left[\frac{2-\omega}{\gamma}\left\{\frac{k_2}{\Omega_0^2}r^{-J_1-2} - \frac{J_3}{J_1+2} - \frac{J_2 J_3\beta^2 r^{\omega-2}}{(\omega-2)(J_1+\omega)\Omega_0^2} + \frac{J_4\beta^2 r^{\omega-2}}{(J_1+\omega)\Omega_0^2}\right\}\right]^{1/2} \exp\left\{\frac{J_2\beta^2 r^{\omega-2}}{2(2-\omega)\Omega_0^2}\right\}} \tag{17}$$

III. RESULTS AND DISCUSSION

Expression (16) and (17), representing, the shock velocity (U) and shock strength (U/a_o) of strong spherical converging shock in a rotating gas in presence of weak magnetic field, shows its dependence on adiabatic index (γ), density parameter (ω propagation distance (r), angular velocity(Ω_o) and constant (ξ).

Taking the initial strength of shock $U/a_o = 1.25$ at r = 2 , ω=1.1, ξ=9, $\Omega_0 = 10$ and $\beta^2 = 0.1$, for γ=1.4, $J_2\beta^2\Omega_0^2 r^{\omega-2}/(2-\omega) = -0.25$, $a'^2 = 41.69937$ and $k_2 = 6947.472636$, the dependence of shock velocity (U) and shock strength(U/a_o), pressure (p/ρ') and particle velocity (u) with propagation distance (r), density parameter (ω), angular velocity(Ω_0), and adiabatic index (γ) are numerically calculated and shown in figures(1-20) and tables-1, respectively.

It is found that the shock velocity (U) and shock strength (U/a_o) both increase as shock converges [cf. Fig. 1-2]. In case of diverging shock, shock velocity initially decreases as shock advances and attains a minimum value at certain propagation distance, thereafter it starts increase[10]. Shock velocity and shock strength both decrease as β^2 increases [cf. Fig. (3-4)]. Shock velocity (U) and shock strength (U/a_o) decreases with increases in density parameter (ω) [cf. Fig. (5-6)]. Shock velocity increase with increase in adiabatic index (γ) while shock strength decreases with increase in γ [cf. Table 1]. It is clear from the figures (7-8), shock velocity and shock strength both decrease, as ξ increases. Shock velocity increases and shock strength decreases with increase in Ω_o [cf. Fig. (9-10)].

Expressions for the pressure and the particle velocity immediately behind the strong spherical converging shock in presence of weak magnetic field**,** respectively, can be written as

$$\frac{p}{\rho'}=\left\{\frac{\chi'\Omega_0^2 r^{2-\omega}}{2-\omega}+\frac{F_1\beta^2 a'^2}{\gamma}\right\}\left[\frac{2-\omega}{\gamma}\left\{\frac{k_2}{\Omega_0^2}r^{-J_1-2}-\frac{J_3}{J_1+2}-\frac{J_2J_3\beta^2 r^{\omega-2}}{(\omega-2)(J_1+\omega)\Omega_0^2}+\frac{J_4\beta^2 r^{\omega-2}}{(J_1+\omega)\Omega_0^2}\right\}\right]\exp\left\{\frac{J_2\beta^2 r^{\omega-2}}{(2-\omega)\Omega_0^2}\right\} \tag{18}$$

$$u=\frac{\xi-1}{\xi}\left[k_2 r^{-J_1}-\frac{J_3 r^2\Omega_0^2}{J_1+2}-\frac{J_2J_3\beta^2 r^{\omega}}{(\omega-2)(J_1+\omega)}+\frac{J_4\beta^2 r^{\omega}}{(J_1+\omega)}\right]^{1/2}\exp\left\{\frac{J_2\beta^2 r^{\omega-2}}{2(2-\omega)\Omega_0^2}\right\} \tag{19}$$

The variation of the pressure (p/ρ') and particle velocity (u) behind the shock at ω = 1.1,1.3 and 1.5 has been given in figure (11-12). It is seen that pressure and particle velocity behind the shock, increase as shock converges. The pressure (p/ρ') and particle velocity (u) decreases as β^2 increases [cf. Fig. (13-14)] and increases with γ [cf. Table 1]. It is clear from the figure (15-16), pressure and particle velocity both decrease with increase in ω and it decrease with decrease in ξ [cf. Fig. (17-18)]. It may be noted from the figures (19-20), respectively, the pressure and particle velocity both increase with angular velocity Ω_o.

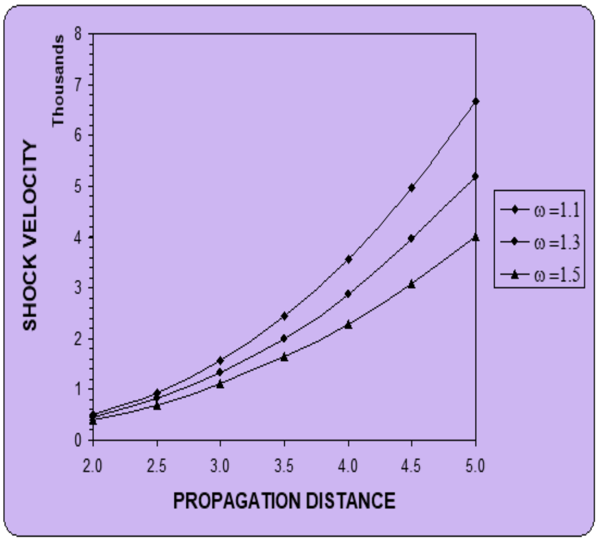

Figure 1: Variation of shock velocity (U) with propagation distance (r) at β^2 = 0.1, ξ = 9, Ω_o = 10 and γ = 1.4.

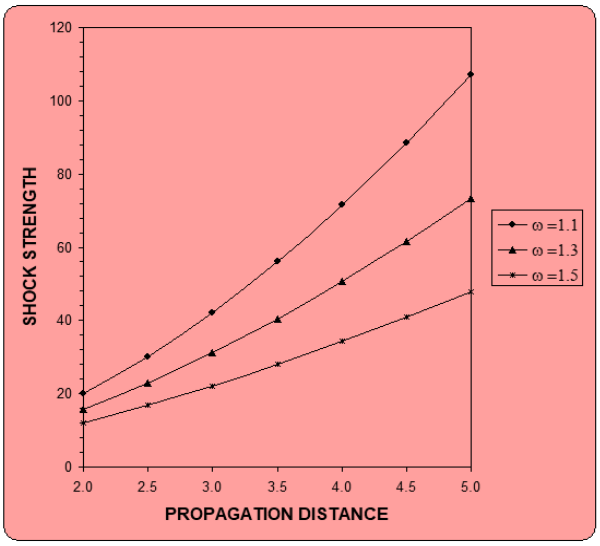

Figure 2: Variation of shock strength (U/a_o) with propagation distance (r) at β^2 = 0.1, ξ = 9, Ω_o = 10 and γ =1.4.

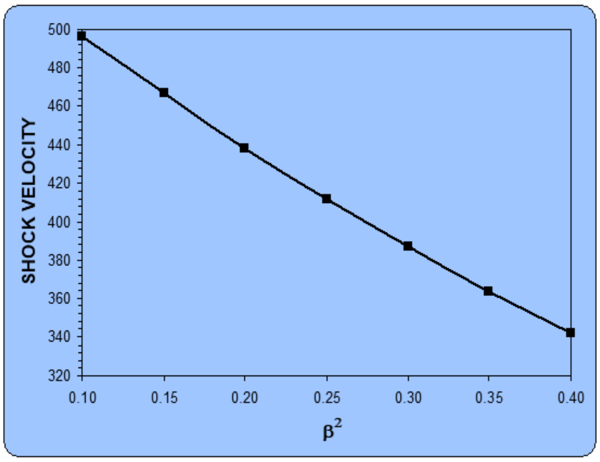

Figure 3: Variation of shock velocity(U) with β^2 at r = 2, ξ = 9, Ω_o = 10 and γ = 1.4.

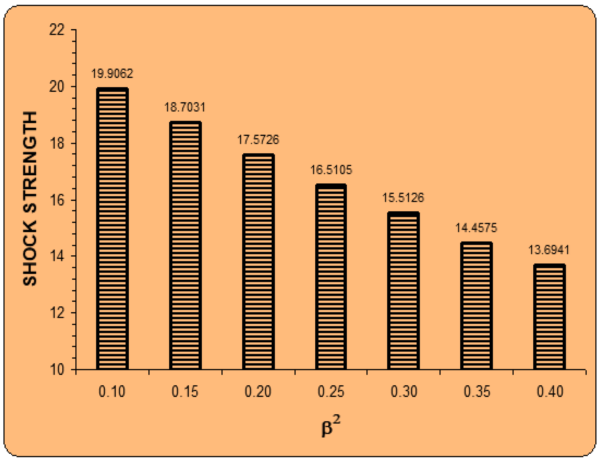

Figure 4: Variation of shock strength (U/a_o) with β^2 at r = 2, ξ = 9, Ω_o = 10 and γ = 1.4.

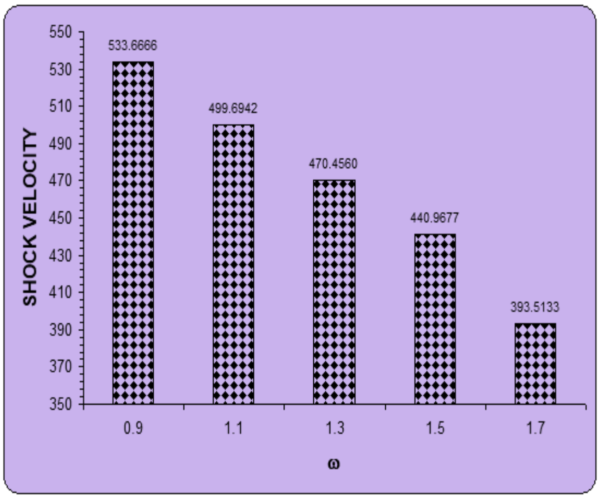

Figure 5: Variation of shock velocity (U) with ω at β^2=0.1, r = 2, ξ = 9, Ω_o = 10 and γ = 1.4.

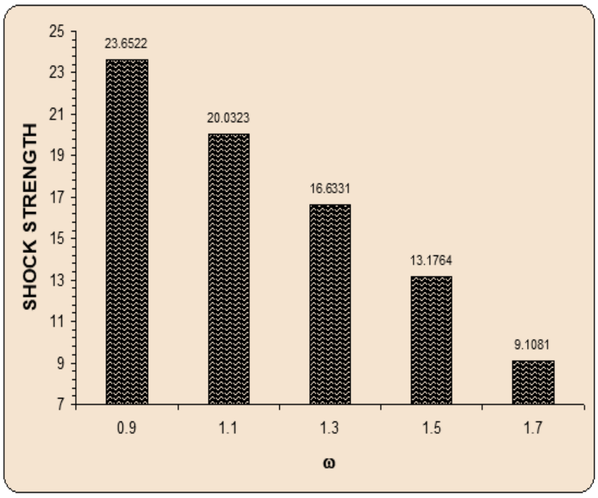

Figure 6: Variation of shock strength (U/a_o) with ω at β^2=0.1, r = 2, ξ = 9, Ω_o = 10 and γ = 1.4.

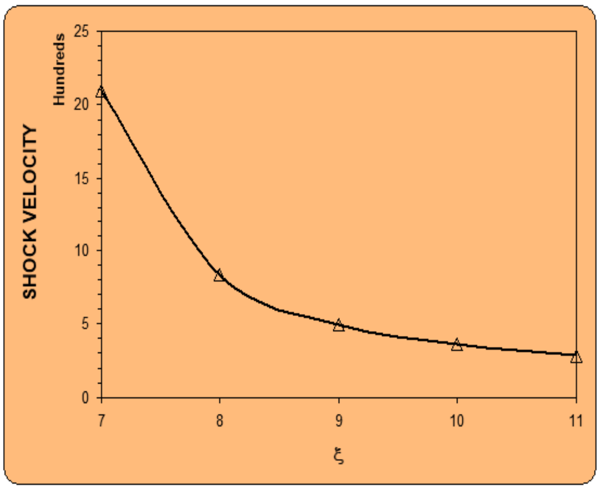

Figure 7: Variation of shock velocity (U) with ξ at ω=1.1, β^2 = 0.1, r = 2, Ω_o = 10 and γ = 1.4

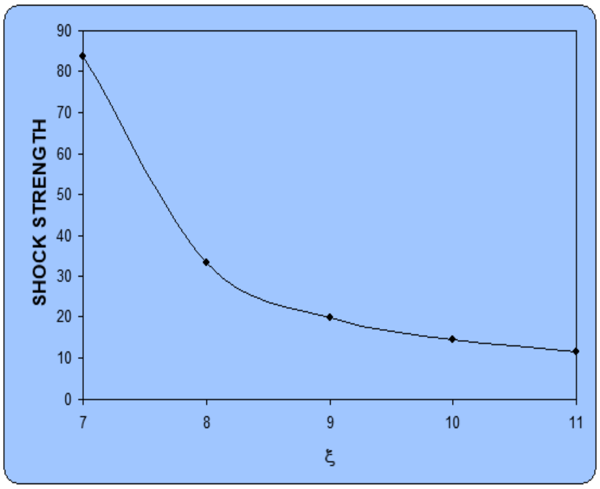

Figure 8: Variation of shock strength (U/a_o) with ξ at ω=1 .1, β2 = 0.1, r = 2, Ωo = 10 and γ = 1.4

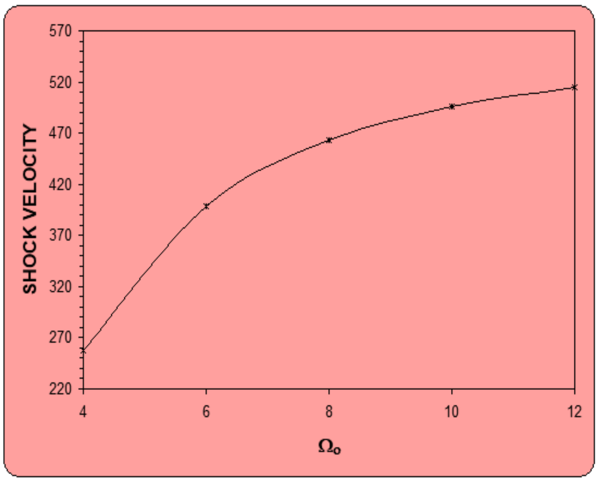

Figure 9: Variation of shock velocity (U) with Ω_o at ξ=9, ω=1.1, β^2 = 0.1, r = 2, and γ = 1.4.

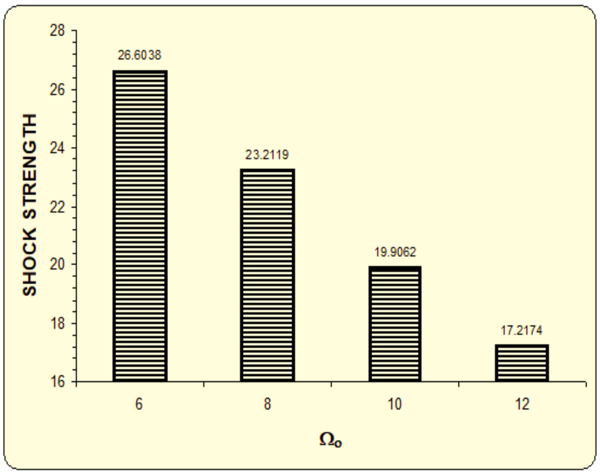

Figure 10: Variation of shock strength (U/a_o) with Ω_o at ξ=9, ω=1.1, β^2 = 0.1, r = 2, and γ = 1.4.

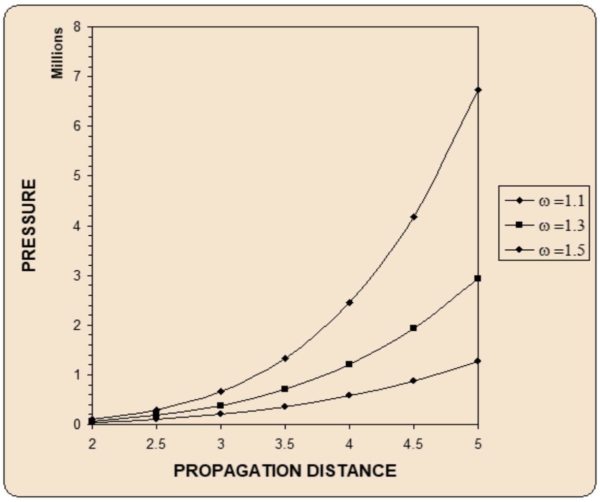

Figure 11: Variation of pressure (p/ρ') with propagation distance (r) at $\beta^2 = 0.1$, $\xi = 9$, $\Omega_o = 10$ and $\gamma = 1.4$.

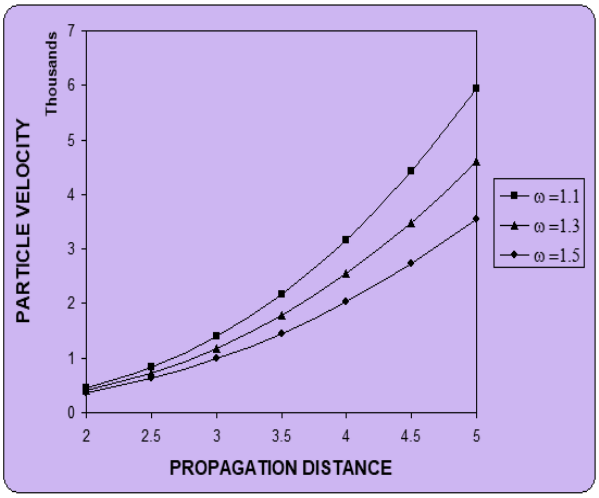

Figure 12: Variation of particle velocity (u) with propagation distance (r) at $\beta^2 = 0.1$, $\xi = 9$, $\Omega_o = 10$ and $\gamma = 1.4$.

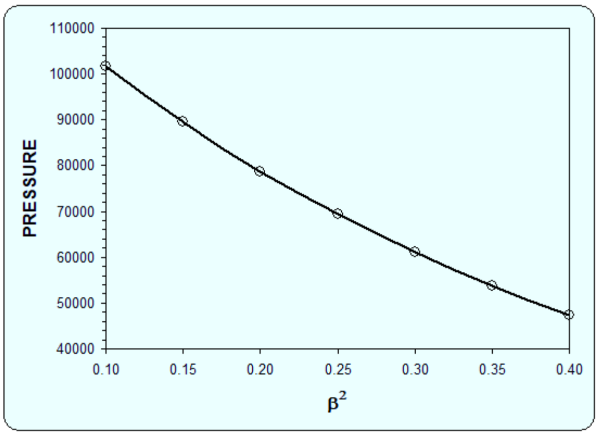

Figure 13: Variation of pressure (p/ρ') with β^2 at $r = 2$, $\xi = 9$, $\Omega_o = 10$ and $\gamma = 1.4$.

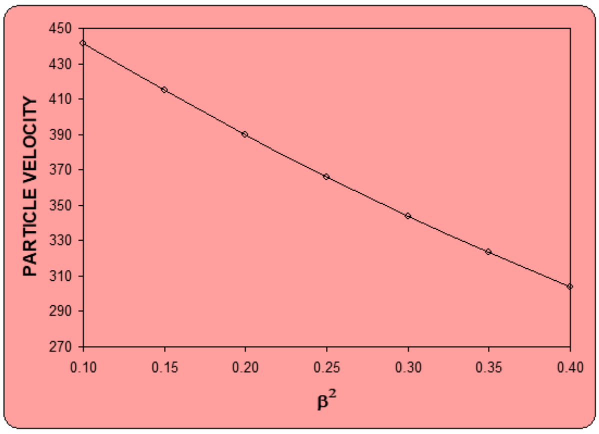

Figure 14: Variation of particle velocity(u) with β^2 at $r = 2$, $\xi = 9$, $\Omega_o = 10$ and $\gamma = 1.4$.

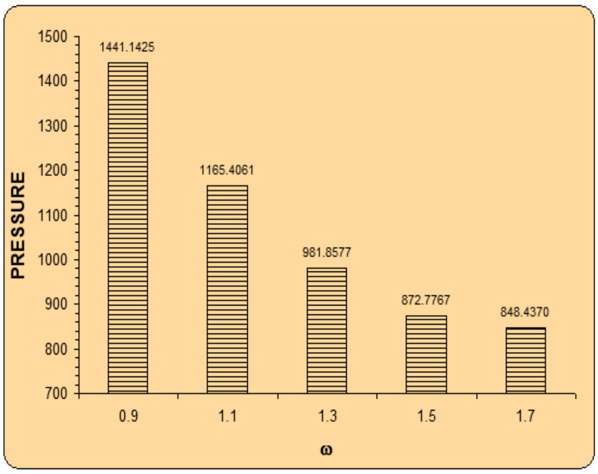

Figure 15: Variation of pressure (p/ρ') with ω at $\beta^2=0.1$, $r = 2$, $\xi = 9$, $\Omega_o = 10$ and $\gamma = 1.4$.

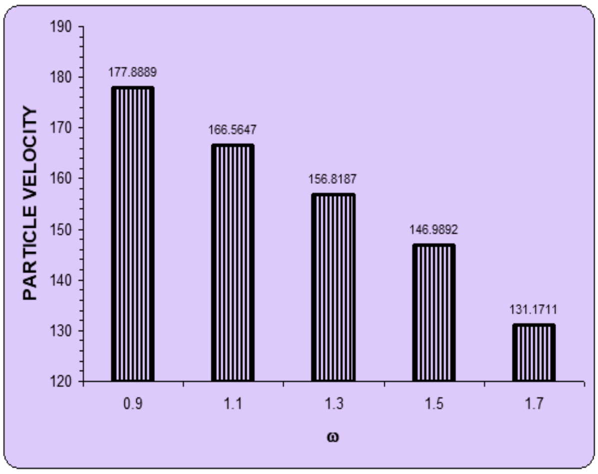

Figure 16: Variation of particle velocity(u) with ω at $\beta^2=0.1$, $r = 2$, $\xi = 9$, $\Omega_o = 10$ and $\gamma = 1.4$.

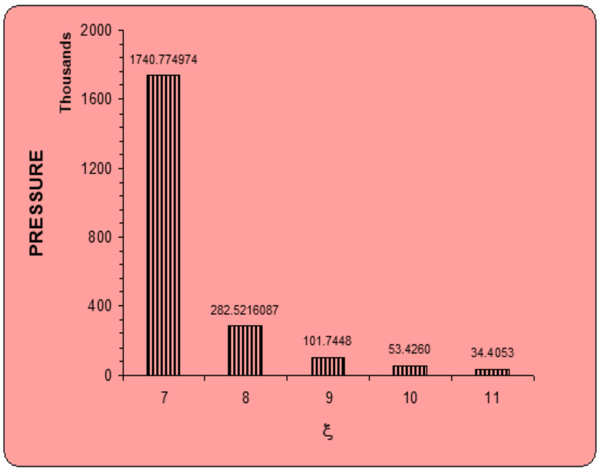

Figure 17: Variation of pressure (p/ρ') with ξ at ω=1.1, β^2 = 0.1, r = 2, Ω_o = 10 and γ = 1.4

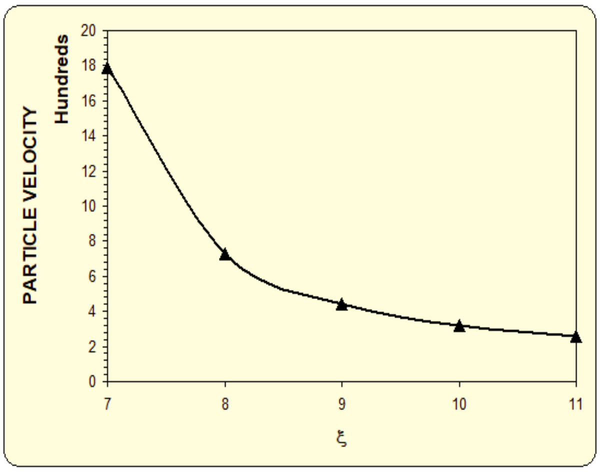

Figure 18: Variation of particle velocity(u) with ξ at ω=1.1, β2 = 0.1, r = 2, Ωo = 10 and γ = 1.4

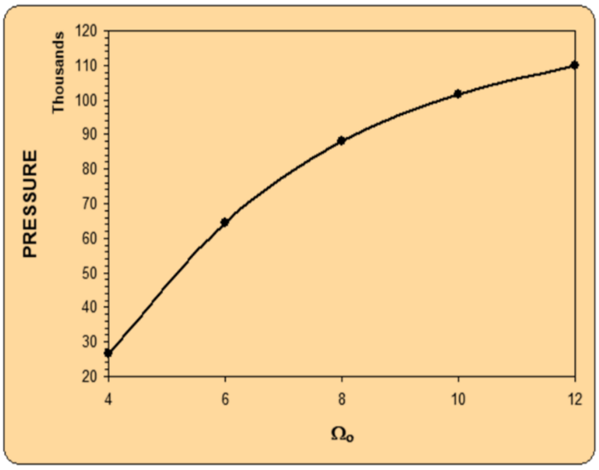

Figure 19: Variation of pressure (p/ρ') with Ω_o at ξ=9, ω=1.1, β^2 = 0.1, r = 2, and γ = 1.4.

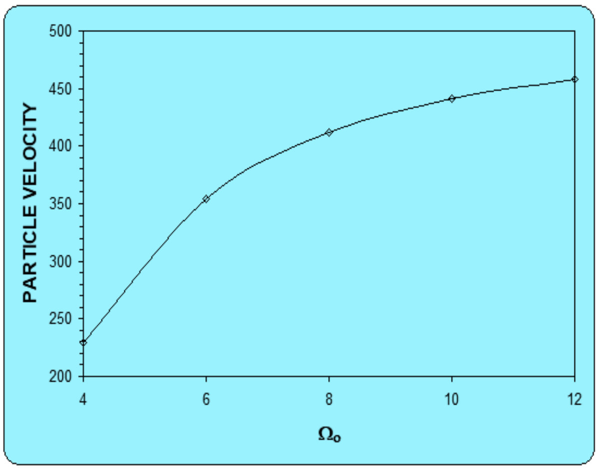

Figure 20: Variation of particle velocity(u) with Ω_o at ξ=9, ω=1.1, β^2 = 0.1, r = 2, and γ = 1.4.

Table 1: Variation of shock velocity (U), shock strength (U/a_o), pressure (p/ρ') and particle velocity (u) with adiabatic index (γ) at ω = 1.1, β^2 = 2, ξ = 1.5, Ω_o = 10 and r = 2 for spherical converging strong shock in weak magnetic field.

ADIABATIC INDEX (γ)	SHOCK VELOCITY (U)	SHOCK STRENGTH (U/a_o)	PRESSURE (p/ρ')	PARTICLE VELOCITY (u)
1.40	496.5481	19.9062	101744.7809	441.3761
1.50	500.8505	19.3990	103562.3827	445.2271
1.67	522.4369	19.1764	112715.1414	464.3884

VI. CONCLUSION

Adiabatic propagation of spherical converging magnetohydrodynamic strong shock waves in rotating gaseous medium has been explained by using well known CCW theory. The effect of overtaking wave behind the shock front has been neglected in this study. Rotation of the medium has been considered as solid body rotation with power varying initial density distribution. Analytical expressions for shock velocity, shock strength, pressure and particle velocity have been derived under weak axial magnetic field. It is found that the presence of magnetic field plays an significant role on all the flow variables. Role of initial angular velocity and density parameter is also discussed through graphs and tables for different adiabatic index of the gas. Finally, the results compared with those for diverging shock.

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