Imploding Detonation Waves in Self-gravitating Ideal Gas

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Abstract: *In the present paper, the propagation of strong spherical imploding detonation waves in a reacting ideal gas under the effect of self-gravitation of the non-homogeneous medium has been investigated, by the help of Chester-Chisnell-Whitham (CCW) theory. It is considered that detonation wave is initially Chapman-Jouguet. Initial taking the density distribution law as power decreasing with distance, the analytical expressions for the detonation velocity just behind the front along with other flow variables are derived. Neglecting the effect of overtaking disturbances the variation of non-dimensional detonation velocity, the pressure and density with the propagation distance have been calculated numerically. The effect of change in density parameter at different Alfven Mach number on the convergence of detonation front have been discussed through graphs in details. Finally, it is found that density parameter and Alfven Mach number of gas have a significant role on propagation of strong spherical detonation front in reacting ideal gas with gravitation effect on all the post-flow variables. The software MATLAB have been used for computation of the problem.*
Key Word*: Self-gravitating gas, Strong detonation waves, CCW theory.*

I. INTRODUCTION

Study of the propagation of detonation waves in reacting gases have great importance in the research field of astrophysics and shock dynamics. Many scientists have considered the problem of motion of detonation waves in different type of homogeneous and non-homogeneous media. Welsh[1], Nigmatulin[2] and Tiepel[3] have assumed the shock wave by a contracting detonation wave front moving into a uniform combustible ideal gas. In magnetogasdynamics, Vishwakarma[4] have extended the problem of Nigmatulin[2]. Verma and Singh[5,6] have investigate the problem of detonation wave propagation of Tiepel[3] in non-homogeneous atmosphere. Chester-Chisnell-Witham (CCW) theory[7–9] have been applied by Tyl and Wlodarczyk[10] for theoretical investigation on the concentric detonation waves in gaseous explosive mixtures. Effect of self-gravitation on the adiabatic propagation of shock waves for weak and strong shock having different type of symmetries in pure and dusty gases have been studied by Gangwar[11–13].

In the present paper, the propagation of imploding detonation waves having spherical symmetry have been studied when the ideal gas under the effect of self-gravitation excluding the effect of flow behind the front. The method of CCW is used to investigate the problem. The detonation is a Chapman-Jouguet front i.e. it. travels with sonic speed relative to the burst gas, which determines the law of convergence. The values of the pressure and internal energy in the undisturbed fluid have been neglected in comparison to their values in the disturbed gas i.e. strong detonation wave. The constant amount of heat is produced during the detonation process and by adding this, the basic flow equations only be corrected.

II. FUNDAMENTAL EQUATIONS, BOUNDARY CONDITIONS AND ANALYTICAL EXPRESSIONS

The Fundamental equations for one dimensional unsteady, adiabatic flow of an ideal reacting gas can be written as Tyle[10,13,14]

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \rho\frac{\partial p}{\partial r} + \frac{Gm}{r^2} = 0 \tag{1}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\rho + \rho a^2\left(\frac{\partial u}{\partial t} + \frac{2u}{r}\right) = 0 \tag{2}$$

$$\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} + \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{3}$$

$$\frac{\partial m}{\partial t} - 4\pi r^2 \rho = 0 \tag{4}$$

where u, p, ρ, m, G and E are the particle velocity, the pressure, density, mass of the radius r, Gravitational constant and internal energy per unit mass of the ideal gas, respectively and 'a' is the local speed of sound in ideal gas is given by

$$a^2 = \gamma p / \rho \tag{5}$$

where γ is the ratio of specific heats of the of the gas. The equation of state for ideal gas is given by

$$p = R_m \rho T \tag{6}$$

where $c_p - c_v = R_m$ is the gas constant, and c_p and c_v are the specific heats of the gas at constant pressure and volume, respectively. T is the temperature of the medium. The internal energy per unit mass of the gas in this case may be given as

$$E = \frac{p}{\rho(\gamma - 1)} \tag{7}$$

The density distribution law of the gas ahead of detonation front is taken to be power varying and given as

$$\rho_a = \rho_k \eta^{-w} \tag{8}$$

where ρ_k is the constant and the $\eta = r_a / R$ is the ratio of radius 'r_a' to the internal radius of the detonation front 'R' and 'w'a positive integer and is known as density parameter.

Let u_a, ρ_a, p_a and E_a represent the undisturbed values of flow velocity, density, pressure, and internal energy per unit mass of ideal gas just ahead of the detonation front, and u, ρ, p, and E be the modified values of respective quantities at any point across the passage of the detonation front. The jump conditions across the strong detonation in ideal gas detonation front in this case may be written as Whitham[9], Tyl and Wlodarczyk[10], Vishwakarma and Vishwakarma [4]

$$\rho(U_D - u) = \rho_a U_D \tag{9}$$

$$p = p_a + \rho_a U_D u_a \tag{10}$$

$$E = E_a + \frac{1}{2}(p + p_a)\left(\frac{1}{\rho_a} - \frac{1}{\rho}\right) + q \tag{11}$$

where U_D and q denote the velocity of detonation front and heat energy released per unit mass, respectively. The indices 'a' refers to the sates just ahead the detonation front.

The detonation front is assumed to be in Chapman-Jouguet state. Chapman-Jouguet condition requires that the flow ahead of the shock front will be in sonic state and, in the shock-fixed coordinates, i.e.

$$\left|U_{D_{CJ}} - u_{CJ}\right| = a_{CJ} \tag{12}$$

where the indices $_{CJ}$ denote the Chapman-Jouguet state.

The boundary conditions across the detonation front in this case are

$$p_{CJ} = \frac{\rho_a}{\gamma + 1} U_{D_{CJ}}^2 \tag{13}$$

$$u_{CJ} = \frac{1}{\gamma + 1} U_{D_{CJ}} \tag{14}$$

$$\rho_{CJ} = \frac{\gamma + 1}{\gamma} \rho_a \tag{15}$$

$$a_{CJ} = \frac{\gamma}{\gamma + 1}\left|U_{D_{CJ}}\right| \tag{16}$$

$$U_{D_{CJ}} = \sqrt{2(\gamma^2 - 1)q} \tag{17}$$

Using the equation sand (8) and (12)-(17) the boundary conditions across the strong detonation front having power decreasing density distribution may be written in the term of velocity $\left(\xi = u_n / u_{CJ}\right)$ of burnt gas

$$\frac{U_D}{U_{D_{CJ}}} = \frac{\xi^2 - 1}{2\xi} \tag{18}$$

$$\frac{p}{p_{CJ}} = \frac{\xi^2 + 1}{2}\eta^{-w} \tag{19}$$

$$\frac{\rho}{\rho_{CJ}} = \frac{\gamma(\xi^2 + 1)}{(\gamma+1)+(\gamma-1)\xi^2}\eta^{-w} \tag{20}$$

$$\frac{a}{a_{CJ}} = \frac{(\gamma-1)\xi^2 + \gamma + 1}{\sqrt{2\gamma[(\gamma+1)+(\gamma-1)\xi^2]}} \tag{21}$$

where $\xi = u_n/u_{CJ}$ and $\eta = r_a/R$,

Under the CJ-condition [15,16] the detonation velocity in terms of Alfven ah number M_{CJ} is given by

$$U_D = U_{D_{CJ}} \quad \text{and} \quad M_{CJ} = U_{D_{CJ}}/a_a \tag{22}$$

where $a_a^2 = \gamma\, p_a/\rho_a$

From equation(4), the mass inside the sphere of radius r is given by

$$m = 4\pi\rho_k\frac{r^{3-w}}{3-w} \tag{23}$$

At the equilibrium state, of the gas is assumed to be specified by the condition, $u = 0 = \partial/\partial t$, $p = p_a$ Therefore, from equation(1), the equilibrium condition prevailing the front of the shock can be written as

$$\frac{1}{\rho_a}\frac{dp_a}{dr} = -\frac{Gm}{r^2} \tag{24}$$

On solving the equation(24) and putting the value of m from equation(23), we have

$$p_a = \frac{2\pi G\rho_k^2}{(3-w)(w-1)}\eta^{2(1-w)} \tag{25}$$

From the above expression (25) it is observed that for the positive initial pressure the value of density parameter is of the range $3 > w > 1$ [17]. The local speed of sound in the ideal gas having self-gravitation effect with power decreasing density distribution can be given by using the equations(5) and (25) we have:

$$a_a^2 = \frac{2\pi\gamma G\rho_k}{(3-w)(w-1)}\eta^{(2-w)} \tag{26}$$

Using the equations(14),(17) and (22), we get

$$u_{CJ}^2 = \frac{2\pi\gamma G\rho_k M_{CJ}^2}{(\gamma+1)^2(3-w)(w-1)}\eta^{(2-w)} \tag{27}$$

The characteristic form of the fundamental basic equations for converging shock *i. e.* the form in which equation contains derivatives in only one direction in (r, t) plane is

$$dp - \rho a\, du + \frac{2\rho a^2 u}{(u-a)}\frac{dr}{r} - \frac{\rho a}{(u-a)}\frac{Gm\, dr}{r^2} = 0 \tag{28}$$

Equation divided by $\rho_{CJ}u_{CJ}^2$ and using the equations after simplifying, we have

$$\frac{d\xi}{d\eta} = \frac{1}{\eta}\frac{(\xi^2+1)}{[\gamma(\xi^2+1)\alpha - \xi]}\left[\frac{w}{2} - \frac{2\alpha\left[\{(\gamma-1)\xi^2 + \gamma + 1\}\alpha\xi\gamma M_{CJ}^2 - (\gamma+1)^2(w-1)\right]}{[\xi - \gamma\{(\gamma-1)\xi^2 + \gamma + 1\}\alpha]\gamma M_{CJ}^2}\right] \tag{29}$$

where $\alpha = [2\gamma\{(\gamma+1)+(\gamma-1)\xi^2\}]^{-1/2}$.

Numerical integrating the differential the equation(29) and using boundary conditions(18)-(20) we get the variation of ξ *i.e.* $\dfrac{u_n}{u_{CJ}}$, $\dfrac{U_D}{U_{D_{CJ}}}$, $\dfrac{p}{p_{CJ}}$ and $\dfrac{\rho}{\rho_{CJ}}$ with propagation distance η.

III. RESULTS AND DISCUSSION

Under the effect of the self-gravitation the expressions for non-dimensional detonation velocity $\frac{U_D}{U_{D_{CJ}}}$, the pressure behind the detonation wave $\frac{p}{p_{CJ}}$, and the density across the detonation front $\frac{\rho}{\rho_{CJ}}$ in terms of the nondimensional flow velocity $\frac{u}{u_{CJ}}$ just behind the strong spherical imploding detonation wave may be obtained by using the equation(18),(19),(20) and(29), respectively. The equation(29) has been solved numerically with the help of software MATLEB and calculate the values of all other post front values of parameters. It is observed that velocity of detonation wave $\frac{U_D}{U_{D_{CJ}}}$, the non-dimensional pressure behind the detonation front $\frac{p}{p_{CJ}}$, and the density across the spherical detonation wave $\frac{\rho}{\rho_{CJ}}$ all flow variables are depend upon the propagation distance $\left(\eta = \frac{r_0}{R}\right)$, adiabatic index of the gas (γ), density parameter(w), and Alfven Mach number M_{CJ}. It is clear from the expressions(29) all the flow parameters are not directly depends upon Gravitational constant G as in simple case of implosion spherical shock wave[17] Initially taking the flow velocity $\frac{u}{u_{CJ}} = 2$ and $\gamma = 1.2$. The variation of detonation velocity $\frac{U_D}{U_{D_{CJ}}}$ with the propagation distance (η), the pressure across the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ for $\gamma = 1.2; w = 2.5, 2.55$ and $M_{CJ} = 2.2, 2.22$ have been computed and displayed through Fig. (1)-(6).

It is important to mention here from the equation(23) and (25) for finite positive equilibrium pressure, the constant w should obey the inequality $1<w<3$[17] It is observed from Fig.(1),(3) and (5) that strong detonation front velocity $\frac{U_D}{U_{D_{CJ}}}$, pressure across the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ is decreases with shock converges in the medium having self-gravitational effect with power decreasing initial density distribution. All parameters are depending upon the value of Alfven Mach number M_{CJ} and also decrease with increase in M_{CJ} from 2.2,2.22 and 2.25. The variation of all flow parameter with propagation distance at different density parameters is shown in Fig.(2),(4) and Fig.(6), it is noticed that the value of density parameter w increases from 2.5 to 2.55 the slope of the graph in increases at fix value $M_{CJ} = 2.2$ and $\gamma = 1.2$. It is also observed that strength of detonation wave is also increase with density parameter (ω). The results obtained here are compared with the results for the problem of propagation of strong spherical shock waves under the effect of gravitation with variable density excluding effect of overtaking disturbances and the similar nature of change in perturbation of flow parameters are observed.[17]
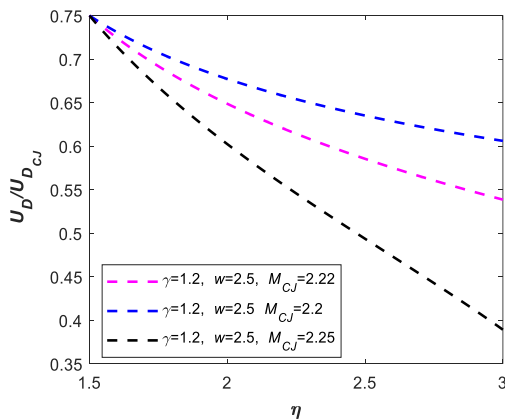

Fig. 1. Variation of detonation front velocity $U_D/U_{D_{CJ}}$ with the propagation distance (η) for $M_{CJ} = 2.2$ and 2.22 at $w=2.5$ and $\gamma=1.2$.
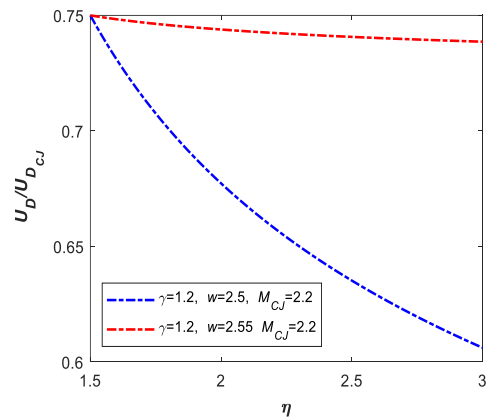
Fig. 2. Variation of detonation front velocity $U_D/U_{D_{CJ}}$ with the propagation distance (η) for $w=2.5$ and 2.55 at $M_{CJ} = 2.2$
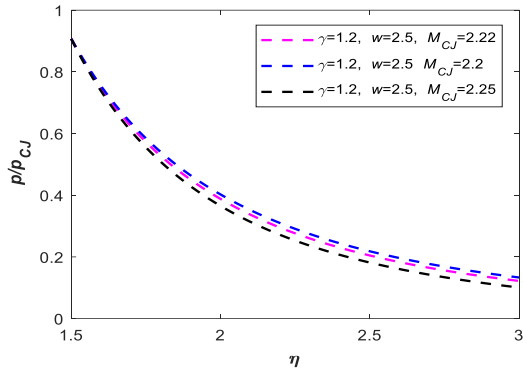
Fig. 3. Variation of pressure p/p_{CJ} with the propagation distance (η) for $M_{CJ}=2.2$ and 2.22 at $w=2.5$ and $\gamma=1.2$.
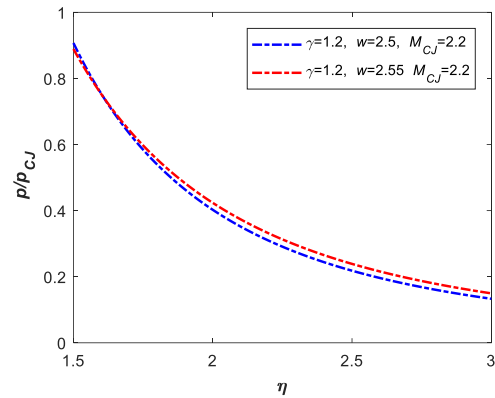

Fig. 4. Variation of pressure p/p_{CJ} with the propagation distance (η) for $w=2.5$ and 2.55 at $M_{CJ}=2.2$
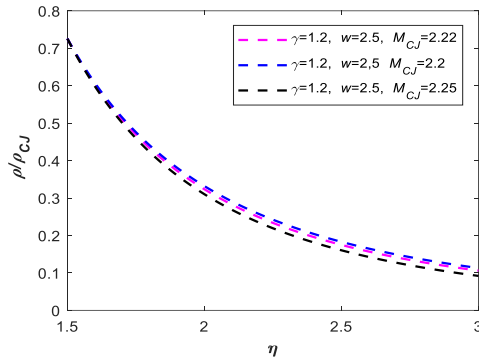

Fig. 5: Variation of density ρ/ρ_{CJ} with the propagation distance (η) for $M_{CJ}=2.2$ and 2.22 at $w=2.5$ and $\gamma=1.2$.
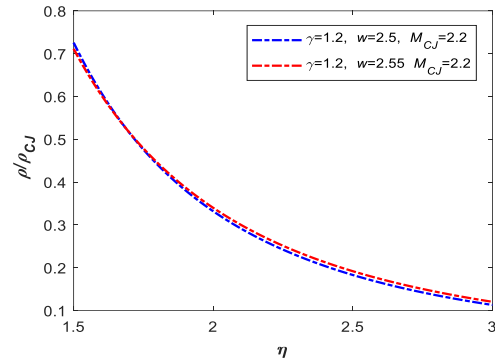

Fig. 5: Variation of density ρ/ρ_{CJ} with the propagation distance (η) for $w=2.5$ and 2.55 at $M_{CJ}=2.2$

III. CONCLUSION

The well-known theory (CCW) of shock dynamics has been applied for the problem of motion of strong spherical detonation wave front under the effect of self-gravitation of the gas with power varying density distribution. We have neglected the effect of the flow behind the detonation front and maintain the CJ state in the study. Perturbation in all the flow variables like detonation velocity pressure and density across the front have been calculated and explained through graphs.

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